PRIVATE VALUATION OF COMPENSATION STOCK OPTIONS

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- **ABSTRACT:** In recent years, risk-adjusted compensation packages with stock options have increase in popularity, volume, and scope. The objective of this paper is to document the divergence between the cost to the firm and the value to the employee of compensations stock options. Simulations reaching numerical examples beyond previous work that halted at theoretical approaches achieve this goal. The cost to the firm and the valuation for a diversified investor would coincide. However, the employee that receives stock options is bearing more firm-related risk that he would under a portfolio optimization strategy. Therefore, the undiversified employee assigns a lower value to the option. The results presented in this paper may help to better understand the preferences for certain types of options over others, from the firm's and from the holder's perspective.
- **KEYWORDS:** Option valuation; executive compensation; risk aversion; non-traditional options.
- ***RESUMEN:*** Recientemente, la popularidad, volumen y alcance de la compensación ajustada al riesgo con opciones sobre acciones ha aumentado. El objetivo de este texto es documentar la divergencia entre el costo para la empresa y el valor que el empleado asigna a la compensación con opciones sobre acciones. Este objetivo se logra mediante simulaciones y ejemplos numéricos superando trabajos previos que se detienen en aproximaciones teóricas. El costo para la empresa y la valoración para un inversionista diversificado coincidirían. Sin embargo, el empleado que recibe opciones sobre acciones asume un riesgo específico mayor que el correspondiente bajo una estrategia de optimización de cartera. Por lo tanto, el empleado no-diversificado asigna

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un valor inferior a la opción. Los resultados presentados ayudarán a un mejor entendimiento de las preferencias hacia determinados tipos de opciones desde las perspectivas de la empresa y del recipiente.

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I. MOTIVATION

Compensation Stock Options (CSO) are used for its potential on achieving recruitment, motivation and retention of talent. Popularity of stock options as part of compensation packages has expanded beyond exclusive executive circles to larger categories of employees. High-tech start-up companies first used CSO as a major part of their compensation packages, but options are also popular among established firms. Chang et al. (2015) analyze the case of extending stock options broadly to non-executive employees. Such options represent substantial claims against the firms and can have an impact on the market value of equity.

Presently, public firms include on their financial statements the cost of CSO at their fair value. Before, options had not to be accounted for on the financial statements. Firms that decided to reveal options on the footnotes had the choice to declare its intrinsic value or its fair value. The intrinsic value is the spread between the stock price at the grant date and the strike price. It was an argument in favor of granting options at the money, since the cost to the firm was apparently nil. Granting options in the money (discount options) would have implied recognition of costs to the firm and immediate taxable income to the recipient (Hall and Murphy, 2000 and 2002). A central point about CSO is the discrepancy between the cost to the firm and the value that the option holder attaches to it. The incentive effects of restricted stocks and options relates to the holder's private valuation.

Extending previous work, this paper explores and quantifies the discrepancy between the cost to the firm and the value to the CSO holder.

The firm's shareholders, that ultimately issue the options, have the freedom to diversify their portfolio investment, as well as any investor that would buy warrants of the firm. The value of options to a diversified investor equals the cost to the firm (their opportunity cost), calculated with traditional option pricing methods. In contrast, executives and employees have an important portion of their total wealth linked to the firm, in the form of wages, firm's stocks and (previously issued) options. CSO are not transferable, and therefore illiquid. Executives are constraint, either by contract or by reputation, to hold a larger number of firm's stocks that would be optimal under a diversified portfolio strategy (Hall and Murphy, 2002). They are also forbidden to short-sell the firm's stock to hedge the risk. Therefore, their valuation of the options corresponds to an undiversified investor. The opportunity cost is the fair value evaluated either by a Black-Scholes (1973) formula (henceforth BS) or by a binomial method. To take into account the propensity to early exercise the options, the expiration date should be replaced with the option's expected life. Chang et al. (2015) also use a BS approach to examine the incentive effects of non-executive employee stock options. The asymmetric structure of stock options payoff rewards long-term success while tolerates short-term failure.

To an undiversified investor, the risk neutrality assumption, key on arbitrage pricing models, binomial models and Montecarlo methodologies, is no longer fulfilled. That may help explaining why CSO are generally exercised as soon as they are vested, prior to maturity, even without dividend payments, which would appear sup-optimal from a diversified investor's perspective (Huddart and Lang, 1996 and Ingersoll, 2002).

Moreover, early exercise decreases the cost to the firm and increases the value to the recipient at the same time (Ingersoll, 2002).

II. STYLIZED FACTS ABOUT COMPENSATION STOCK OPTIONS

Because of the diversity of interpretations in the literature, a precision is in order. The options are award on the grant date, and in general have maturity up to ten years. Many firms allow their executives to exercise options prior to expiration (American type options), but options become exercisable only after a vesting period, and if termination or retirement occurs before the vesting period, the options are usually forgone. Options remain exercisable until expiration, and if the options are in the money after the vesting period, the holder may decide to exercise them, and he can receive:

a) A cash amount equivalent to the difference between the current stock price and the strike price, if the firm has a paid in cash policy. No new shares are issued.
b) Restricted stocks, in exchange of the payment of the strike price that would eventually vest themselves at their own vesting date. In this case the stock will have a private value at the option's exercise date, and an adjustment for the remaining vesting period must be made, which amounts to a discount to the stock price. Ingersoll (2002) shows that the subjective option value is reduced about 3% further in this case. This case can be seen as an American option on restricted stocks.
c) Stocks and new options with a higher strike price. The strike price is paid with (restricted) stocks already held by the investor and valued at its market value. Since the options are in the money when

exercised, the number of stocks is less than the number of options. In exchange of the original options, the investor receives a number of stocks equal to the number of original options and a number of new options equal to the number of shares tendered (smaller than the original number of options), but with a higher strike price equal to the current stock price (the original options are in the money and the new options are at the money), and same maturity than the original options. Johnson and Tian (2000) analyze this type of Reload Options. The number of new shares effectively issued by the firm is smaller than otherwise, and employee share ownership is stimulated.

d) Unrestricted stocks, in exchange of the payment of the strike price, which he can decide to sell at the market price, or to keep. When the option is exercised, the strike price is added to paid-in capital and the number of shares increases. This is the general case analyzed here.

Some firms issue options that vest according to a pre-established plan. With cliff vesting, all options granted on a given date vest after a set period of time. With straight vesting, options vest gradually over time; the same proportion vests each year. For example 33% of the options vest annually over 3 years, 25% vest annually over 4 years or 20% vest annually over 5 years (Hall and Murphy 2002). With stepped vesting, a different proportion vest each year. For example, 10%, 20%, 30% and 40% of the options might vest each year (Ingersoll, 2002). The value of the compensation package, results on a weighted average of options with different vesting periods. Moreover, by early exercising, the option holder can invest the proceeds in more profitable or less risky assets.

Carpenter (1998) indicates that to value the options, the writer should determine the exercise policy of the option holders. Some models of optimal exercise policy for undiversified executives demonstrate that with sufficiently high-risk aversion and low wealth, the options should be exercised as soon as they vest (or gets in the money). However, Carpenter (1998) shows that executives hold options long enough and deep enough in the money before exercising to capture additional value.

The fair value should be evaluated using the options' expected life instead of the expiration date; however some authors criticize such approach. In addition it is suggested using the historical volatility, which is one of the major objections in modern option pricing to BS methods. Kulatilaka and Marcus (1994) claim that historical data are of limited use because historical exercise experience is subject to past stock returns. They argue that the stock volatility and the expected term of the option should not be chosen independently, for exercise policy depends on the path of the stock price. They suggest that CSO should be valued using Monte Carlo simulations like in the mortgage industry. The vesting period, the period after which the options can be exercised is typically from three to five years (Huddart and Lang, 1996). Moreover, most CSO are exercised as soon as they vest, so ten-year options serve to align incentives for only five years, when exercised early. Therefore, in this paper it is assumed that CSO are European and that the relevant horizon is the vesting period of five years (see Carpenter 1998). Using vesting periods as exercise dates also helps to reduce forfeiture complications that would need information about departure probabilities (employee turnover). These assumptions allow for comparison between the cost to the firm and the value to the options holder under the same conditions. There is a limited body of research on executive

compensation outside the US, thus a theoretical approach supported by simulations is of universal application and interest.

Executive contracts face a number of constraints (cognitive, social-psycological, informational and incentive-compatibility[*]) that make them incomplete. Executive compensation reflects such limitations probably resulting from suboptimal bargaining rather than "arms lengths" negotiation. However, Ferri and Maber (2013) find support for the argument that the levels and growth of CEO pay could be mostly the result of market forces, which supports the efficient contracting view that CEO pay properly reflects the value of managerial skills. Essen, Otten and Carberry (2015) conduct a Meta-analysis over 219 studies to explore support for the managerial power theory. They describe a process between relatively powerful boards and relatively powerful CEOs in which negotiations about executive compensation is driven by board structures and shareholders characteristics. They conclude that CEOs influence their own compensation arrangements.

Incentives and Compensation

From a compensation perspective, CSO have some virtues and some defects as well. They are supposed to align firm's performance and executive's wealth, by encouraging them to take actions that increase stock price. Options are however inefficient (expensive). The cost to the firm is much more than the value the holder recognizes, because of exercise restrictions (vesting periods), illiquidity and forfeiture. Such cost-value discrepancy has been long time ignored. On the other hand, lifting the restrictions will cause the CSO to lose all their potential for motivation and

[*] Friendship, loyalty, collegiality, the desire to build cohesion in the board room, and expanding shareholders rights vs. avoiding reputation penalties in the directors labour market. In addition, uncertainty about CEO compensation could reduce the supply of managerial talent to publicly listed firms.

retention. Without vesting restrictions they would be equivalent to a cash bonus, which is more likely compensating for past performance than securing future commitment. In addition, transferable options would not benefit from deferred taxability for recipients. Martin, Wiseman and Gomez-Mejia (2013) suggest the use of proper benchmarks (rival firms) before stock options become exercisable. As Carpenter (1998) points out, tax advantages from delaying exercise may offset the benefits of diversification. The requirement that the employees leaving the firm forfeit their options (with exceptions) helps also explaining the tendency to early exercise. For the same reasons, the common practice to reprice underwater CSO (options that went out of the money because the stock price plunged since the grant date) does not help to achieve those objectives either. Repriceable Options are criticized because repricing equals to forgive executives for past performance. However, it is also argued that underwater options have little probability of maturing at the money and therefore, repricing restores some of its incentive and retention potential (Ingersoll, 2002). In addition, Hall and Murphy (2002) find that refraining from repricing underwater options is not necessarily in the interest of the firm. In the same line, Ferri and Maber (2013) analyze the after match of the adoption in 2002 of a legislation that mandates a non-binding shareholder vote on executive compensation ("say-on-pay"). They find an increase in the sensitivity of CEO pay to performance. They document a reduction on rewards for failure: (option repricing, severance packages .i.e. golden parachutes).

The following analysis includes all categories of executives and employees. The main differences between the two groups are the asymmetry of information, the level of wealth and therefore their level of risk aversion. For a private valuation perspective it boils down just to using different

values on some parameters. However, André, Boyer and Gagné (2001) find that the CEO exercises his option sooner than other executives.

Strike price

Hall and Murphy (2000) find that to the firm the range of optimal exercise price includes always the grant date stock price (at the money options). They claim that there is little loss in terms of incentive compared to accounting charges in granting at the money options instead of discount options. They show that the optimal exercise price is lower for large grants, and for less diversified and more risk adverse executives. From the executive's perspective the optimal exercise price is the lowest possible. An option with cero exercise price would be just a restricted stock.

Warrant valuation

Warrant valuation differs from option valuation in that the former accounts for the dilution effects of issuing new shares when the warrant is exercised (Gallai and Schneller, 1978). Kulatilaka and Marcus (1994) argue that for its small effects and for simplicity dilution may be just ignored. Dilution is dismissed in this paper as well because the focus of the analysis is on the divergence between the cost to the firm and the value to the holder, so the dilution effects are likely to affect both sides in the same proportion. However, it should probably be included when the cost to the firm is reported on the financial statements.

III. LITERATURE REVIEW

Johnson and Tian (2000) argue that firms choose from a large menu of traditional and non- traditional options to design executive compensation packages. They use risk-neutral valuation principles to provide close-form

solutions to the cost to the firm for a number of so-called non-traditional European stock options. With risk-neutral valuation it is assumed that options are redundant securities which payoff can be replicated by dynamically trading on the underlying asset and the risk free asset. Johnson and Tian (2000) indicate that non-traditional stock options have different impact, than traditional options, on parameters that are under the influence of the executives of the firm, such as the stock price, the stock return volatility and the dividend yield. Among the options they present are the Premium Option (out of the money options), the Performance Vested Option, the Repriceable Option and the Purchase Option, which analytical values may be seen as linear combinations of BS formulas. The BS approach is used in this case as one of the possible approaches to value stock options. Certainly, all the options analyzed could be modeled using other methodologies. Alternative methods would include binomial o trinomial trees as well as simulations by Montecarlo techniques. However adding methodological complexity would only obscure the goals this paper is trying to achieve here. Moreover, international accounting standards (FASB, 2004) that require that executive stock options are priced and expensed at fair price accept BS methodologies. Similarly, widely used databases such as ExecuComp yield results based on BS methods. We focus on popular exotic options that have close form solutions to introduce a measure of risk aversion.

The Premium Option is granted out of the money. Hall and Murphy (2002) find that granting options out of the money on the grant date is not necessarily in the interest of the firm. Indeed, firms rarely award Premium Options.

The Performance Vested Option comes to existence if the stock price reaches certain value (barrier option up and in). The stock price needs just to hit the barrier once; it is not required to remain above the barrier for a minimum of days, which could be a plausible requirement. It can be seen as a linear combination of three traditional BS options. The first option has a strike price equal to the (up) barrier level (B_u). The second option has a strike price equal to the stock price at the grant date, and the current stock price is replaced by the square of the barrier level divided by the stock price. The third option has a strike price equal to the barrier level, and the current stock price is replaced by the square of the barrier level divided by the stock price. The appendix A details the weighting coefficient of the three options and the additional term.

Firms may agree with the holders to alter the terms of the options, if the shareholders permit. The Repriceable Option has the advantage for the holder that the strike price may be reset to a lower level if the stock price plunges and reaches a low barrier level (B_d). It is assumed that the repricing can be done once only, which does not need to be the case. The value of the Repriceable Option is calculated assuming a barrier level rather than assuming a repricing date. The Repriceable Option may be seen as the sum of two barrier options, a down and up option and a down and in option, or alternatively as the linear combination of three options. The first option has a strike price equal to the stock price at the grant date. The second option has a strike price equal to the barrier level, and the current stock price is replaced by the square of the barrier level divided by the stock price. The third option has a strike price equal to the stock price at the grant date, and the current stock price is replaced by the square of the barrier level divided by the stock price. One difference between the Performance Vested Option

and the Repriceable Option is that in the first case the barrier is above the stock price at the grant date and in the second case the barrier level is below. A second difference is on the factors of the linear combination of the three options in each case as can be seen in Appendix A. Ju, Leland and Senbet (2014) analyze a lookback call option that would be similar and superior to a repriceabe option.

The Purchase Option requires the holder to pay a non-refundable fraction of the strike price the date it is granted. The option has a strike price equal to the stock price at the grant date reduced by the prepaid fraction. The prepaid fraction is then reduced from the BS value of the option. Johnson and Tian (2000) indicate that just few firms use Purchase Options.

Table N°1
Cost to the firm. Scenarios for initial stock prices.

Stock Price	Stock Volatility	Traditional Option	Premium Option	Performance Vested Option	Repriceable Option	Purchase Option
100	10%	37,15	18,31	35,35	37,15	31,51
100	20%	40,35	26,6	39,80	41,21	33,78
100	30%	45,60	34,99	45,41	48,61	38,22
90	10%	33,44	16,48	31,81	33,44	28,36
90	20%	36,32	23,94	35,82	37,09	30,40
90	30%	41,04	31,49	40,87	43,75	34,40
110	10%	40,87	20,14	38,88	40,87	34,67
110	20%	44,39	29,26	43,78	45,33	37,16
110	30%	50,16	38,49	49,95	53,47	42,04

Table # 1 shows the values of the options for different stock prices on the grant date, for the same BS parameters as in Johnson and Tian (2000) Table 1, page 14. The exercise price is 100, the stock price is 100, the dividend yield is 2%, the risk free rate 8% and the time to expiration is 10 years. The stock volatility is 0.1, 0.2 and 0.3. The up barrier is 150, the down

barrier is 50, and the prepaid portion is 10% of the strike price 100. The strike price is set equal to the stock price on the grant date, the up barrier is one and a half times the stock price on the grant date, the down barrier is half the stock price on the grant date and the prepayment is ten percent of the strike stock price on the grant date. Table # 1 may help to decide the timing of granting CSO from the firm's perspective.

Value to the undiversified CSO holder

Ingersoll (2002) demonstrates that for an undiversified CSO holder the value of the option may be calculate with a modified BS formula that takes into account his risk aversion and diversification restrictions. He derives a model for the marginal value of options, under the same conditions that the BS model. By contrast, Martin, Wiseman and Gomez-Mejia (2013) use a particular methodological approach. They claim that in the money options have a value that represents current wealth. They compute the current wealth as the spread between the stock price and the strike, which is only the option value one instant before exercising it. However, fair option value at any time computed by BS or binomial methods is indeed the present value of expected payoffs. In a call option the payoff may be positive or null. Since most options are issued at the money Martin, Wiseman and Gomez-Mejia (2013) approach would suggest that option have not value when grated, which is rejected by standard accounting and taxation practices. Since, they do not formally acknowledge other initial endowment, there is not incentive to protect current wealth. Similarly, to compute prospective wealth Martin, Wiseman and Gomez-Mejia (2013) abstract from established methods and suggest a spread between the current stock price and a future stock price increased by a compound grow rate. Again, BS and binomial

methods include the likelihood of each outcome to occur. With just prospective wealth at the stake, there are incentives to increase firm risk.

Ingersoll's (2002) model can be used to evaluate heterogeneous options, which mature on different sates and can also be used each time a new option is granted. Ingersoll (2002) solves the investor's consumption-investment problem, using a standard continuous-time framework with a constraint opportunity set. The investor has a power utility function defined over lifetime consumption: $U = C^{\gamma}$ (constant relative risk aversion coefficient CRRA = $1-\gamma>0$). The model determines the subjective value assigned to the option due to the holder's lack of diversification and risk aversion. From a different perspective, Martin, Wiseman and Gomez-Mejia (2013) contrast classic agency theory and behavioural agent theory assuming that the CEO is loss averse instead of risk averse. They support a mixed game in which the CEO trades-off current equity wealth and prospective wealth. They also discuss the consequences that arise when CEOs are permitted to hedge against the idiosyncratic firm risk. They analyze hedging as an example of active risk management. Their global results should be interpreted as additional evidence that executive stock options lose the incentive strength as maturity approaches. However, stock options approaching maturity in the money have already fulfilled their objectives. Ingersoll (2002) assumes that the continuous–time CAPM holds, so the efficient portfolio is the market portfolio. Until retirement, the investor (manager or employee of the firm) must hold a fraction α of his wealth in his firm's stock (beyond that represented in the market portfolio). Before retirement, the option holder invests on the risk free asset, the market portfolio, and the firm's stock. The subjective interest rate is lower than the actual interest rate because of the relative risk aversion γ, the stock–holding constraint α and the residual

variance υ^2. Therefore, a certain future payment has present value to the constraint investor higher than its market value, because such present value would have to be invested sub-optimally to a lower subjective interest rate. Similarly the subjective discount rate for the market portfolio is lower than the objective discount rate predicted by the Capital Asset Pricing Model (CAPM) and the subjective discount rate for the firm's stock is higher than the market-unconstrained rate. After retirement, as long as the financial market is perfect and the CAPM holds, the optimal portfolio strategy is to hold the risk free asset and the market portfolio, for the firm's stock is already represented in the market portfolio. Then, the solutions of the maximization utility are the optimal consumption and portfolio choices as given by Merton (1996). Utility is always higher for the unconstrained problem (see Appendix B for formulations).

Ju, Leland and Senbet (2014) examine the effects of stock option on corporate risky investments. They suggest an optimal combination of fixed payments, stocks and stock options that minimizes the total cost to the firm. They claim that the direct cost to the firm of stock options is small when compared to agency costs of suboptimal investment. They favour restricted stock rather than stock options since additional call option makes the portfolio riskier. They also note that stock options are generally issued at the money because of accounting and tax considerations. At the same time, the small size of the variable portion of the executive's compensation creates incentives to increase risk.

Hedging for undiversified investors

Carpenter (1998) argues that an undiversified CSO holder can always hedge by selling short stocks (or an index) that are highly correlated with his firm's stock. Cao and Wei (2004) use a continuous-time, consumption-

portfolio framework to demonstrate that a hedging index can alleviate the dead weight loss created by the liquidity and vesting restrictions of the CSO and restricted stocks as well, while preserving retention and long-time incentive effects. Cao and Wei (2004) extend the work of Ingersoll (2002) augmenting the portfolio choice set consisting of the market portfolio (M), the firm's stock (S) and the risk free asset (B) to include a hedging index (I). The hedging index can be an industry index with a high correlation to the firm's stock. Cao and Wei (2004) find that the deadweight loss associated with options is generally much larger than that associated with restricted stocks. The larger loss is primarily due to the non-linear nature of the option's payoff. They find also that the hedging index is much more effective in reducing the deadweight loss of CSO compared to restricted options. The employee obtains the highest utility when the shorting restriction is absent. When the index is not included in the portfolio the utility is the lowest and corresponds to the simple portfolio choice set as in Ingersoll (2002). The levels of utility obtained translate into the valuation of the CSO and the reduction of the deadweight loss. Cao and Wei (2004) find that even constraint hedging is better than not hedging. The private valuation of the option can be expressed using a BS formula with interest rate and dividend yield parameters replaced by the discount rate and the illiquidity discount respectively, to adjust for subjective valuation. Since the illiquidity discount positive and the discount rate is smaller than the dividend yield, the subjective value of the option is less than its market value. Cao and Wei (2004) proof that the role of the index is non-trivial. The illiquidity discount and the excess variance are reduced when the index is present, therefore the hedging index will narrow the gap between the private valuation and the market valuation (see Appendix C for details on the hedging index).

IV. APPLICATIONS.

The first analysis consists to apply Ingersoll's (2002) modified Black and Scholes formula, for European option valuation to some of the alternative options discussed by Johnson and Tian (2000). Ingersoll (2002) suggests the extension of his model to handle the modifications seen at incentive options. In Table # 2, the value to the option holder and the cost to the firm are calculated for each type of option. This permits a complete set of comparisons, for reasonable values of parameters. The constant relative risk aversion coefficient CRRA ranges from 1 to 7; and the stock holding (diversification) parameter α ranges from 10% to 75% of total wealth. The BS parameters have been uniformized to be consistent with the following sections, previous work and actual stylized facts in the literature (Fama and French, 2001 and 2002). The first row "Unrestricted" shows the cost of the compensation policy the firm will report. Taking into account risk aversion could help to explain behavioural deviance from expected alignment between the interest of the shareholders and the executives. Thus, long-term value creation could be a goal not shared by all. The results may help to better understand the preferences for certain types of options over others, from the firm's and from the holder's perspective. Ju, Leland and Senbet (2014) define a risk-averse manager as someone that is willing to sacrifice a higher current stock price for lower future uncertainty.

Table N°2

Value to an undiversified investor of several non-traditional CSO.

CRRA coefficient	Stock Holding (in excess)	Traditional Option	Premium Option	Performance Vested Option	Repriceable Option	Purchase Option
	α					
Unrestricted (0%)		35,96	20,80	35,13	38,34	30,21
1	10%	33,29	18,90	32,48	35,71	27,38
	25%	30,00	16,55	29,19	32,49	23,90
	50%	25,98	13,64	25,17	28,66	19,69
	75%	23,33	11,64	22,50	26,32	16,98
3	10%	28,41	15,53	27,62	30,88	22,18
	25%	20,25	10,10	19,52	22,84	13,42
	50%	11,88	5,02	11,27	14,70	4,29
	75%	7,17	2,53	6,67	10,21	0,00
5	10%	24,09	12,65	23,33	26,58	17,53
	25%	13,07	5,85	12,47	15,60	5,50
	50%	4,44	1,47	4,10	6,71	0,00
	75%	1,34	0,32	1,19	3,02	0,00
7	10%	20,28	10,22	19,57	22,77	13,40
	25%	8,03	3,20	7,57	10,35	0,00
	50%	1,32	0,33	1,18	2,71	0,00
	75%	0,14	0,02	0,12	0,63	0,00

Stock price 100, Strike price 100, stock volatility 30%, υ 27,5%, T =5 years, dividend yield 0%. Rf 5%.

Negative values for Purchase Option with the higher values of risk aversion and lack of diversification parameters are set to cero.

In Table # 3, the value to the option holder is compared to the cost to the firm for each type of option. The second row compares the unrestricted cost to the firm of a non-traditional option with the traditional option. From the option holder's perspective, the Repriceable Option will be preferred, to any other option, including the traditional option, but it is too expensive to the firm. From the firm's perspective, the Premium Option represents a good choice, it is the less costly and at the same time the decline

in value to the option holder is less dramatic. The Purchase Option would never be chosen, it is relatively inexpensive, but its incentive effects are inferior that those of any other option, and really unacceptable for high risk-averse and highly undiversified investors. For high values of risk aversion and lack of diversification all the options lost their attractiveness.

Table N°3

Value to an undiversified investor, of non-traditional CSO, compared to the respective unrestricted option (value vs. cost).

CRRA coefficient	Stock Holding (in excess)	Traditional Option	Premium Option	Performance Vested Option	Repriceable Option	Purchase Option
	α					
Unrestricted (0%)		35,96	20,8	35,13	38,34	30,21
		100%	58%	98%	107%	84%
1	10%	92,6%	90,9%	92,5%	93,1%	90,7%
	25%	83,4%	79,6%	83,1%	84,7%	79,1%
	50%	72,2%	65,6%	71,6%	74,8%	65,2%
	75%	64,9%	56,0%	64,0%	68,7%	56,2%
3	10%	79,0%	74,6%	78,6%	80,5%	73,4%
	25%	56,3%	48,6%	55,6%	59,6%	44,4%
	50%	33,0%	24,1%	32,1%	38,3%	14,2%
	75%	19,9%	12,1%	19,0%	26,6%	0,0%
5	10%	67,0%	60,8%	66,4%	69,3%	58,0%
	25%	36,3%	28,1%	35,5%	40,7%	18,2%
	50%	12,3%	7,1%	11,7%	17,5%	0,0%
	75%	3,7%	1,5%	3,4%	7,9%	0,0%
7	10%	56,4%	49,1%	55,7%	59,4%	44,4%
	25%	22,3%	15,4%	21,6%	27,0%	0,0%
	50%	3,7%	1,6%	3,4%	7,1%	0,0%
	75%	0,4%	0,1%	0,3%	1,6%	0,0%

Stock price 100, Strike price 100, stock volatility 30%, υ 27,5%, T =5 years, dividend yield 0%. Rf 5%.

Table # 4 may be of interest to the holder of non-traditional options. It permits to estimate how much worse-off or better-off he is with respect

to holders of traditional option, or holders of different non-traditional options, for a reasonable set of values on the parameters. The stock holding (diversification) parameter is 10% and 25% of total wealth.

Table N°4

Choice of instrument: value to an undiversified investor of several non-traditional CSO, compared to the traditional option.

CRRA	Stock Holding (in excess)	Traditional Option	Premium Option	Performance Vested Option	Repriceable Option	Purchase Option
	α					
Unrestricted (0%)		35,96	20,8	35,13	38,34	30,21
		100%	58%	98%	107%	84%
		1,00	1,73	1,02	0,94	1,19
1	10%	100%	56,8%	97,5%	107,2%	82,2%
	25%	100%	55,2%	97,3%	108,3%	79,7%
3	10%	100%	54,6%	97,2%	108,7%	78,1%
	25%	100%	49,9%	96,4%	112,8%	66,3%
5	10%	100%	52,5%	96,9%	110,3%	72,8%
	25%	100%	44,7%	95,4%	119,4%	42,1%
7	10%	100%	50,4%	96,5%	112,3%	66,1%
	25%	100%	39,8%	94,3%	128,9%	0,0%

Stock price 100, Strike price 100, stock volatility 30%, υ 27,5%, T =5 years, dividend yield 0%. Rf 5%.

Table # 4 should be interpreted carefully. The recipient of a Premium Option, should compare his private valuation to the cost to the firms, as in Table # 3, and not to the private valuation of another option recipient. When the compensation package is negotiated the firm should offer a number of options that makes the total cost of the options equal in any circumstance. The third row of Table # 4 indicates that the firm is indifferent to offer one traditional option, 1.73 Premium Options, 1.02 Performance Vested Options, 0.94 Repriceable Options or 1.19 Purchase

Options. The private valuation of option holders is always inferior to the cost to the firm. As his risk aversion increases and diversification decreases, his private valuation deteriorates.

Private valuation with a hedging index

The second analysis consists to extend Cao and Wei's (2004) option valuation to some of the alternative options discussed by Johnson & Tian (2000). Cao and Wei (2004) is itself an extension of Ingersoll (2002) where the Black-Scholes formula for European options is modified to include a hedging index that will alleviate the deadweight loss imposed by the vesting restrictions, lack of diversification and illiquidity of CSO. In Table # 5, the value to the option holder is calculated for each type of option. The constant relative risk aversion coefficient CRRA values are 1, 3 and 5; and the stock holding (diversification) parameter α ranges from 10% to 75% of total wealth. Ju, Leland and Senbet (2014) also assume a constant relative risk aversion utility function and calibrate the risk-aversion coefficient with observed data in their simulations. The private valuation is calculated under three circumstances: unrestricted hedging, restricted hedging and no hedging. The no hedging situation is just the case in Ingersoll (2002). As Cao and Wei (2004) noticed the hedging index help to alleviate the deadweight loss, improving the private valuation while the cost to the firm remains constant. The wealth of the option holder is still linked to the performance of the firm; therefore the incentive effect of the compensation packages should remain intact. When the discrepancy between the value to the option holder and the cost to the firm become finally accepted, some firms will help their own employees and executives to find an appropriate hedging index and find a way to convince brokers to reduce shorting restrictions to accommodate optimal hedging.

Table N°5

Private valuation of an undiversified investor of several non-traditional CSO, with different shorting restrictions on the hedging index. %.

	Unrestricted hedging	Restricted hedging	No hedging	Unrestricted hedging	Restricted hedging	No hedging	Unrestricted hedging	Restricted hedging	No hedging
	CRRA = 1			CRRA = 3			CRRA = 5		
Traditional Option									
10%	34,80	34,02	33,29	32,56	30,38	28,41	30,43	27,05	24,09
25%	33,31	31,52	30,00	28,44	23,85	20,25	24,09	17,65	13,07
50%	31,44	28,24	25,98	23,49	16,33	11,88	17,00	8,57	4,44
75%	30,22	25,86	23,33	20,26	11,48	7,17	12,58	4,03	1,34
Premium Option									
10%	19,97	19,42	18,90	18,39	16,88	15,53	16,91	14,62	12,65
25%	18,89	17,63	16,55	15,47	12,43	10,10	12,56	8,54	5,85
50%	17,47	15,26	13,64	11,99	7,60	5,02	7,92	3,38	1,47
75%	16,44	13,48	11,64	9,65	4,73	2,53	5,18	1,27	0,32
Performance Vested Option (Barrier Option up & in)									
10%	34,08	33,36	32,68	32,18	30,13	28,23	30,55	27,30	24,35
25%	32,78	31,09	29,61	29,14	24,61	20,78	26,70	19,94	14,40
50%	31,21	28,12	25,73	26,19	18,52	12,71	22,99	12,68	5,62
75%	30,17	25,86	22,92	23,80	13,94	7,46	17,75	6,82	1,60
Repriceable Option (down % up + down & in)									
10%	37,09	36,25	35,47	34,68	32,33	30,21	32,39	28,73	25,56
25%	35,53	33,60	32,00	30,34	25,39	21,60	25,70	18,76	13,92
50%	33,67	30,25	27,98	25,38	17,62	13,06	18,53	9,30	4,97
75%	32,65	28,01	25,73	22,60	12,88	8,75	14,61	4,70	1,90
Purchase Option									
10%	28,98	28,15	27,38	26,61	24,28	22,18	24,34	20,71	17,53
25%	27,42	25,51	23,90	22,24	17,30	13,42	17,58	10,56	5,50
50%	25,47	22,07	19,69	17,03	9,19	4,29	9,98	0,47	0,00
75%	24,26	19,60	16,98	13,68	3,88	0,00	5,21	0,00	0,00

Stock price 100, Strike price 100, stock volatility 30%, υ_s 27,5%, T =5 years, dividend yield 0%. Rf 5%, σ_I 0,25, ρ_{ms} 0,4, ρ_{mI} 0,5, ρ_{Is} 0,8, ρ_{Ism} 0,756.

Table # 6 shows the gain in private valuation with respect to the no hedging situation for a number of scenarios, with reasonable values on the risk aversion and diversification parameters. The constant relative risk aversion coefficient CRRA values are: 1, 3 and 5; and the stock holding (diversification) parameter α is 10% and 25% of total wealth. The recipient of a given option should compare his private valuation with and without hedging restrictions for the same set of values on the others parameters.

Therefore, Table # 6 should also be interpreted carefully. The restricted hedging situation implies that the broker imposes a limit to short selling the index that is half the optimal short position on the index. The restricted hedging is always binding.

Table N°6

Gain in private valuation when the hedging index is included in the portfolio

	Unrestricted hedging	Restricted hedging	Unrestricted hedging	Restricted hedging	Unrestricted hedging	Restricted hedging
		CRRA = 1		CRRA = 3		CRRA = 5
Traditional Option						
10%	4,52%	2,18%	14,61%	6,95%	26,36%	12,30%
25%	11,04%	5,06%	40,45%	17,78%	84,37%	35,08%
Premium Option						
10%	5,64%	2,73%	18,44%	8,73%	33,67%	15,54%
25%	14,11%	6,51%	53,21%	23,11%	114,89%	46,13%
Performance Vested Option (Barrier Option up & in)						
10%	4,29%	2,09%	14,00%	6,73%	25,45%	12,10%
25%	10,69%	4,99%	40,24%	18,46%	85,45%	38,53%
Repriceable Option (down % up + down & in)						
10%	4,57%	2,20%	14,79%	7,00%	26,72%	12,40%
25%	11,02%	4,99%	40,47%	17,58%	84,65%	34,80%
Purchase Option						
10%	5,82%	2,81%	19,97%	9,50%	38,88%	18,17%
25%	14,69%	6,72%	65,75%	28,95%	219,52%	91,91%

Stock price 100, Strike price 100, stock volatility 30%, υ_s 27,5%, T =5 years, dividend yield 0%. Rf 5%, σ_I 0,25, ρ_{ms} 0,4, ρ_{mI} 0,5, ρ_{Is} 0,8, ρ_{Ism} 0,756.

Extensions

Other studies about CSO deal also with restricted stocks (Cao and Wei, 2004; Kahl, Liu, and Longstaff, 2003), American options (Cao and Wei, 2004; Ingersoll, 2002), and indexed options (Cao & Wei, 2004; Ingersoll, 2002; Johnson and Tian, 2000). The variation of employee's option valuation

with other dimensions such as length of the vesting period and volatility of the stock has already been analyzed in previous work in the literature. Some studies also include the incentive effects (Ingersoll, 2002; Johnson and Tian, 2000) estimated as the value of Delta, the derivate of the option price or values with respect to the stock price. In a BS formula Delta is given by the value of N(d1). In this paper the approach is to compare the private value to the holder with the cost to the firm, rather than calculate a theoretical incentive effect. The rationale is that the private valuation of the option is much more intuitive than its first derivative for any option holder. Other "Greeks" has also been studied as they related to incentive effects. Chang et al. (2015) analyze the sensitivity of stock options value to stock price (Delta-performance based incentives) and to stock volatility (Vega-risk taking incentives). Although they do not model risk aversion directly, they address the issue of human capital tied to firm performance controlling by previous stock ownership.

V. CONCLUDING REMARKS

The divergence between the cost to the firm and the value to the employee, of compensations stock options is a research topic that is far from been exhausted. With the accounting rules that make mandatory to expense options, some firms may reduce the grant of options, restrict it to executives that may actually have an impact on value creation or migrate to restricted stocks. However, there is an important amount of equity claim on existing options that makes it an implausible endangered species. It is important to recall that CEO compensation generally includes a mix of fix and variable components. The stock options analyzed here are just one of these components, and by no means have we suggested that all compensation is delivered or should be delivered in such format. Firm risk

related compensation packages do not only increase in popularity over the past decade, but also they increase in complexity and scope. This paper merges and extends previous work for few non-traditional employee stock options. The work of Ingersoll (2002) and the work of Cao and Wei (2004) are applied to some of the options in Johnson and Tian (2000): the Premium Option, the Performance Vested Option, the Repriceable Option, and the Purchase Option. For the above-mentioned options, close form solutions are available, which results from linear combinations of traditional Black-Scholes formulas. The employee that receives stock options is bearing more firm-related risk that he would under a portfolio optimization strategy. A diversified investor would optimally distribute his wealth into the risk free rate and the market portfolio. The cost to the firm and the valuation for a diversified investor would coincide. However, the undiversified employee assigns a lower value to the option. The results presented in this paper may help to better understand the preferences for certain types of options over others, from the firm's and from the holder's perspective. The divergence between the cost to the firm and the private valuation requires that the options provide strong incentives effects to executives and employees to increase the firm value. Additionally, the undiversified investor may reduce his firm-related risk by short-selling an index that is highly correlated to the firms stock, while maintaining the incentive efect of the options.

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APPENDIX A.

The Performance Vested Option is a weighed combination of 3 options. The coefficient of the first option is one. The coefficient of the second option is $(B_u/S_o)^v$ and the coefficient of the third option is $-(B_u/S_o)^v$, where S_o is the stock price on the grant date, $v= 2(r-\delta)/\sigma^2-1$, r is the risk-free rate, δ is the continuous dividend yield and σ is the instantaneous volatility rate of the stock price. An additional fourth term is added: $(B_u-S_o)e^{-rt}N(d_2)-(B_u/S_o)^v(B_u-S_o)e^{-rt}N(d_{2a})$, where N( ) is the cumulative probability function of the standard normal distribution, $d_2=(\ln(S_o/B_u)+(r-\delta-0.5\sigma^2)t)/(\text{sqrt}(\sigma^2t))$, $d_{2a}=(\ln(B_u/S_o)+(r-\delta-0.5\sigma^2)t)/(\text{sqrt}(\sigma^2t))$, and t is the exercise date.

For the Repriceable Option, the coefficient of the first option is also one. The coefficient of the second option is $(B_d/S_o)^v$, and the coefficient of the third option is $-(B_d/S_o)^v$. No additional terms are added.

APPENDIX B.

Ingersoll (2002) uses Ito's lemma to find a partial differential equation for the subjective value of the stock option: $0=0.5\sigma^2S^2F_{ss}+(r\text{^}-q\text{^})SF_s-r\text{^}F+F_t$. Where F =F(S,t) denotes the subjective value of the option, F_{ss}, and F_s are the second and first partial derivatives with respect to the stock price (S), and F_t is the partial derivative with respect to time (t). The solution to the partial differential equation is a form of the BS formula with interest rate and dividend yield parameters adjusted for subjective valuation. The evolution of the market portfolio follows the process: $dM/M=(\mu_m-q_m)dt+\sigma_m d\omega_m$ and the stock price follows the process: $dS/S=(\mu-q)dt+\beta\sigma_m d\omega_m+\upsilon d\omega$, where the Wiener process $d\omega_m$ governs the movement of the market portfolio, the Wienner process $d\omega$ is

the idiosyncratic risk of the company's stock, and υ^2 is the residual variance. The two Wiener processes are independent, so the covariance between the stock and the market is fully captured by β (the total risk of the stock is $\sigma^2=\beta^2\sigma_m{}^2+\upsilon^2$). The subjective value of a CSO is determined as if the dividend yield was larger and the interest rate smaller than they truly are, in the BS formula amended by Merton (1973) to account for proportional dividends. The subjective interest rate is r^=r-(1-γ)$\alpha^2\upsilon^2$, and q^=q+(1-γ)α(1-α)υ^2 is the subjective adjustment to the dividend yield. Since q^>q and r^<r, the subjective value of the option is less than its market value. Both larger dividends and lower interest rate induce call option holders to exercise their options sooner.

APPENDIX C: Hedging for undiversified investors

The preferences of the risk-averse employee are described by a constant relative risk aversion utility function $U = e^{-\phi t}C^{(1-\gamma)}/(1-\gamma)$ (coefficient = $\gamma>0$). As in Ingersoll (2002) the employee is required to hold a fixed fraction of his total wealth in the firm's stock during the vesting period. The introduction of the hedging index helps to reduce the additional non–systematic risk imposed to the employee by the liquidity and transferability restrictions and improves his utility. The price dynamics are $dM/M=(\mu_m-q_m)dt+\sigma_m dz_m$, $dS/S=(\mu_s-q_s)dt+\sigma_s dz_s$, and $dI/I=(\mu_I-q_I)dt+\sigma_I dz_I$. The correlation coefficient between z_m and z_s is ρ_{ms}, the correlation coefficient between z_m and z_I is ρ_{mI}, and the correlation coefficient between z_I and z_s is ρ_{Is}. The residual or partial correlation between the restricted stock and the index is $\rho_{Ism}=(\rho_{Is}-\rho_{ms}\rho_{mI})/\text{sqrt}[(1-\rho_{ms}{}^2)(1-\rho_{mI}{}^2)]$ after controlling for the market impact. The cum-dividend expected returns are u_m, $u_s=r+\beta_s(u_m-r)$, and $u_I=r+\beta_I(u_m-r)$, where $\beta_s=\rho_{ms}\sigma_s/\sigma_m$, $\beta_I=\rho_{mI}\sigma_I/\sigma_m$, and the volatilities are σ_m, σ_s, and σ_I. The non–systematic variance for the stock is $\upsilon_s{}^2=(1-\rho_{ms}{}^2)\sigma_s{}^2$ and for the index is $\upsilon_I{}^2=(1-\sigma_{mI}{}^2)\sigma_I{}^2$. The dividend yields are q_m, q_s,

and q_I, and the percentages of total wealth invested in the risky assets are x_m, x_s, and x_I respectively. With no trading restrictions the solution to the employee's maximization expected utility is $x_m=(\mu_m-r)/\gamma\sigma_m^2$, $x_s=x_I=0$, as in Merton (1969). When the trading restriction on the stock is removed, there is no need to take a position on the index. If the employee is constraint to hold a fixed percentage of his wealth on the firm's stock then $x_s >0$. Given x_s the employee optimizes his portfolio strategy on the market and the index. When there are no trading limits on the index $x_{I*}=-x_s\rho_{Ism}\upsilon_s/\upsilon_I$ and the excess consumption variance is $\Omega=x_s^2\upsilon_s^2(1-\rho_{Ism}^2)$. When the employee faces shorting restrictions imposed by his broker, $x_I<-x_s\rho_{Ism}\upsilon_s/\upsilon_I$ and $\Omega=x_s^2\upsilon_s^2+x_I^2\upsilon_I^2+2\rho_{Ism}x_sx_I\upsilon_s\upsilon_I$.